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D6-branes and M-theory geometrical transitions from gauged supergravity

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Abstract: We study the supergravity duals of supersymmetric theories arising in the world-volume of D6-branes wrapping holomorphic two-cycles and special lagrangian three-cycles within the framework of eight-dimensional gauged supergravity. When uplifted to 11d, our solutions represent M-theory on the background of, respectively, the small resolution of the conifold and a manifold with G_2 holonomy. We further discuss on the flop and other possible geometrical transitions and its implications.

Keywords: M-Theory, D-branes, String Duality, AdS/CFT Correspondance.

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Contents

1. Introduction

The world-volume low-energy dynamics of D-branes in certain curved backgrounds defines a topologically twisted supersymmetric field theory [1]. The twisting is necessary to allow for the world-volume of the brane to support covariantly constant spinors (this is reminiscent of a similar phenomenon in lower-dimensional supergravities [2]). If the D-brane is wrapping a non-trivial cycle, and we take its size to zero, the infrared dynamics of the system is described by a lower-dimensional field theory with either *ordinary* or *twisted* (depending on the higher-dimensional twisting being, respectively *partial* or *full*) *reduced* supersymmetry [3]. The amount of supersymmetry preserved has to do with the way in which the cycle is embedded in a higher-dimensional space. If the number of branes is taken to be large, this sort of systems provide a supergravity dual description of supersymmetric field theories with less than sixteen supercharges [4]–[8].

In this paper we will consider D6-brane configurations that reduce, at low energies, to theories with four and eight supercharges in four and five dimensions. The D6-brane system is best described in the infrared by means of $\mathcal{N} = 2$ seven-dimensional super Yang-Mills theory [9]. So, for example, wrapping these branes on S^3 would imply, after appropriate twisting, breaking one quarter of the supercharges, the theory reducing to $\mathcal{N} = 1$ four-dimensional super Yang-Mills in the infrared. The above referred twisting corresponds to S^3 being a special lagrangian submanifold of a Calabi-Yau threefold, namely, the deformed conifold T^*S^3. On the other hand, if the D6-branes wrap a holomorphic S^2 in the cotangent bundle of S^2, T^*S^2, the infrared dynamics will be governed by five-dimensional $\mathcal{N} = 2$ super Yang-Mills theory.

It was recently proposed that the configuration of D6-branes wrapping an S^3 in T^*S^3 is dual, through a conifold transition, to a type-IIA geometry where the D6-branes have dissapeared, being replaced by RR fluxes on the blownup S^2 [10]. Conversely, there is a mirrored type-IIB version of this phenomenon with D5-branes wrapping the S^2 becoming RR fluxes on the S^3. It was almost immediately realized that this duality can be better viewed in M-theory on G_2 holonomy manifolds [11], where it corresponds to a flop transition [12]. (See also [13] for a recent discussion on topology change in G_2 holonomy manifolds.) It is natural to analyze these configurations in 11d for the fact that uplifted D6-branes become purely gravitational. Besides, the D6-branes are strongly coupled in the ultraviolet and the would be decoupling limit has to be addressed in eleven dimensions. In particular, the 11d supergravity solution is trustable for any number of branes. Another difference with other D-branes is given by the fact that massive geodesics can escape to infinity signaling the non decoupling of gravity [9].

It is our purpose in this paper to study this sort of solutions under the light of lower-dimensional gauged supergravity. This is the natural framework to perform twisting. The solutions emerging from these theories correspond to near horizon D-brane solutions thus giving directly the gravity duals of gauge theories living on the world-volume of the brane. Since we will work with D6-branes, the twisting would require to impose boundary conditions on eight-dimensional gravitational, gauge and scalar fields so, following the methods introduced in [4], the natural set up for this problem is eight-dimensional gauged supergravity. In particular, we will work within the framework of *maximal* 8*d* gauged supergravity [14] so as to have enough room for different twistings. The virtue of gauged supergravities in this respect is that they provide quite cleanly the gauge field modes that undertake the partial twisting.

Uplifting to eleven dimensions will leave us with M-theory on Ricci flat backgrounds corresponding to the small resolution of the conifold and a G_2 holonomy manifold. Both manifolds eventually develop singularities where transitions to a different manifold might be possible. In the latter case, for example, the geometrical transition correspond to the above referred flop between two three-spheres that, at the singular point, constitute the base of a cone [12]. Instead, in the former case, we found that there is no transition, and the theory in the ultraviolet falls into the singularity. The reason for the absence of a geometrical transition can be attributed, as we will discuss, to the non existence of a θ angle in five-dimensional theories. This suggest that a duality between large-N five-dimensional $\mathcal{N} = 2$ super Yang-Mills theory and superstrings propagating in a K3 manifold with fluxes turned on, in the spirit of [10], might not take place.

The plan of the paper is as follows. In section 2 we review maximal gauged supergravity in eight dimensions and prepare the set up for the search of solutions. Section 3 is devoted to the case of D6-branes wrapping special lagrangian 3-cycles. We first construct solutions of 8d gauged supergravity that are subsequently uplifted to 11d. The resulting geometry is that of a G_2 holonomy manifold recently studied in [12]. In section 4 we consider the case of D6-branes on holomorphic 2-cycles in a deformed A_1 singularity of K3, namely T^*S^2. When uplifted to 11d our solution is the small resolution of the conifold $\mathcal{O}(-1) + \mathcal{O}(-1) \to \mathbb{P}^1$. We discuss on the obstructions to the geometrical transitions appearing in this case and their relation to generic aspects of five-dimensional gauge theories. We conclude in section 5 with a discussion of our results, and an outlook of avenues for further research.

Note Added. While the final version of this paper was being typewritten, some results that overlap part of ours were reported by Jaume Gomis [15].

2. Review of $d = 8$ gauged supergravity

Maximal gauged supergravity in eight dimensions was originally constructed by Salam and Sezgin [14]. It arises from dimensional reduction of $11d$ supergravity on a SU(2) group manifold [16]. The field content of this theory consists of the metric $g_{\mu\nu}$, a dilatonic scalar Φ, five scalars given by a unimodular 3×3 matrix L^i_α in the coset $\mathrm{SL}(3,\mathbb{R})/\mathrm{SO}(3)$, a seventh scalar B, a three-form $B_{(3)}$, three two-forms $B^i_{(2)}$, three vector fields $B^i_{(1)}$ and a SU(2) gauge potential A^i_μ, as well as the pseudo Majorana spinors ψ_μ and χ_i. In this paper we are going to restrict ourselves to a sector of the theory with vanishing B-fields. This amounts to pure gravitational solutions of the $11d$ system. The bosonic dynamics in this sector is governed by the lagrangian

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{1}{4}e^{2\Phi}F_{\mu\nu}^{\ i}F^{\mu\nu i} - \frac{1}{4}P_{\mu ij}P^{\mu ij} - \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{g^2}{16}e^{-2\Phi}\left(T_{ij}T^{ij} - \frac{1}{2}T^2\right), \quad (2.1)$$

where e is the determinant of the *achtbein* e^a_μ, $F^i_{\mu\nu}$ is the Yang-Mills field strength and $P_{\mu ij}$ is a symmetric and traceless quantity defined by

$$P_{\mu ij} + Q_{\mu ij} \equiv L^\alpha_i(\partial_\mu\delta_\alpha^{\ \beta} - g\epsilon_{\alpha\beta\gamma}A^\gamma_\mu)L_{\beta j}\,, \quad (2.2)$$

$Q_{\mu ij}$ being the antisymmetric counterpart. We have set $\kappa = 1$. As usual, greek indices are curved ($\alpha, \beta, \ldots$ are in the group manifold[1] and $\mu, \nu, \ldots$ label space-time coordinates) while latin ones are flat. Notice that, for example, $A^\gamma_\mu = L^\gamma_i A^i_\mu$, as well as $F_{\mu\nu}^{\ \alpha} = L^\alpha_i F^i_{\mu\nu}$. Finally, the potential energy corresponding to the scalar fields is governed by the so-called T-tensor,

$$T^{ij} = L^i_\alpha L^j_\beta \delta^{\alpha\beta}\,, \quad (2.3)$$

[1] While working in eight dimensions, these indices describe a flat space. The dependence on the coordinates of the group manifold have been factored out, and it will only reappear when uplifting to $11d$ is performed.

and $T = \delta_{ij}T^{ij}$. The equations of motion are

$$R_{\mu\nu} = P_{\mu ij}P_{\nu}^{ij} + 2\partial_\mu\Phi\partial_\nu\Phi + 2e^{2\Phi}F^i_{\mu\gamma}F_\nu{}^{\gamma i} - \frac{1}{3}g_{\mu\nu}\nabla^2\Phi\,, \tag{2.4}$$

$$\nabla_\mu(e^{2\Phi}F^{\mu\nu i}) = -e^{2\Phi}P_\mu^{ij}F_j^{\mu\nu} - gg^{\nu\gamma}\epsilon^{ijk}P_{\gamma jl}T_k{}^l\,, \tag{2.5}$$

$$\nabla_\mu P^{\mu ij} = -\frac{2}{3}\delta^{ij}\nabla^2\Phi + e^{2\Phi}F^i_{\mu\nu}F^{\mu\nu j} + \frac{g^2}{2}e^{-2\Phi}\Theta^{ij}\,, \tag{2.6}$$

where Θ^{ij} is short for

$$\Theta^{ij} \equiv T^i{}_kT^{jk} - \frac{1}{2}TT^{ij} - \frac{1}{2}\delta^{ij}\left(T_{kl}T^{kl} - \frac{1}{2}T^2\right). \tag{2.7}$$

Notice that the dilaton equation is obtained from (2.6) by tracing over the latin indices.

The supersymmetry transformations for the fermions are given by

$$\delta\psi_\gamma = \mathcal{D}_\gamma\epsilon + \frac{1}{24}e^\Phi F^i_{\mu\nu}\Gamma_i\left(\Gamma_\gamma{}^{\mu\nu} - 10\delta_\gamma{}^\mu\Gamma^\nu\right)\epsilon - \frac{g}{288}e^{-\Phi}\epsilon_{ijk}\Gamma^{ijk}\Gamma_\gamma T\epsilon\,, \tag{2.8}$$

$$\begin{aligned}\delta\chi_i =& \frac{1}{2}\left(P_{\mu ij} + \frac{2}{3}\delta_{ij}\partial_\mu\Phi\right)\Gamma^j\Gamma^\mu\epsilon - \frac{1}{4}e^\Phi F_{\mu\nu i}\Gamma^{\mu\nu}\epsilon - \\ &- \frac{g}{8}e^{-\Phi}\left(T_{ij} - \frac{1}{2}\delta_{ij}T\right)\epsilon^{jkl}\Gamma_{kl}\epsilon\,,\end{aligned} \tag{2.9}$$

where the covariant derivative is

$$\mathcal{D}_\mu\epsilon = \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\Gamma_{ab} + \frac{1}{4}Q_{\mu ij}\Gamma^{ij}\right)\epsilon\,. \tag{2.10}$$

It is useful for later purposes to work alternatively with spinors of 32 components or doublets of sixteen components. Then, we will use the following representation for the Clifford algebra

$$\Gamma^a = \gamma^a \times \mathbb{I}\,, \qquad \Gamma^i = \gamma_9 \times \sigma^i\,, \tag{2.11}$$

where γ^a are eight-dimensional gamma matrices (a being a flat index), $\gamma_9 = i\gamma^0\gamma^1 \ldots \gamma^7$, with $\gamma_9^2 = 1$, and σ^i are the Pauli matrices corresponding to the R-symmetry group. It will be finally convenient to introduce $\Gamma_9 \equiv \frac{1}{6i}\epsilon_{ijk}\Gamma^{ijk} = \gamma_9 \times \mathbb{I}$.

In the following we will consider supergravity duals of gauge theories in four and five dimensions with four and eight supercharges, respectively. Our procedure is based on taking the low energy limit for a D6-brane wrapped on three and two supersymmetric cycles in Calabi-Yau and K3 manifolds. The structure group of the normal bundle of these cycles is, respectively, SO(3) and SO(2), thus Salam-Sezgin theory has enough room for their twisting. When the energies are low enough, the cycle decouples and we remain with a theory that has less dimensions and less supersymmetries than the original one.

Since we will work with D6-branes, it seems natural to consider seven-dimensional boundary conditions for gauge and scalar fields, so, the natural set up for this problem is eight-dimensional gauged supergravity. We can see that the vacuum supersymmetric solution of this theory is given by [17]

$$\begin{aligned} ds_8^2 &= e^{\frac{2}{3}\phi} dx_{1,6}^2 + dr^2 \,, \\ e^{\phi-\phi_0} &= r \,, \end{aligned} \tag{2.12}$$

where $\phi_0 = \log(3g/8)$. When uplifted to eleven dimensions by means of the prescription given in ref. [14], after appropriate coordinate rescaling, the higher-dimensional configuration is

$$ds^2 = dx_{1,6}^2 + N \left(d\rho^2 + \rho^2 d\Omega_3 \right) . \tag{2.13}$$

After modding out the outer three-sphere by $\mathbb{Z}_N$, we get an ALE space with an A_{N-1} singularity in coincidence with the uplifting of the near horizon solution corresponding to D6-branes in type IIA [9].

3. D6-branes on the deformed conifold

In this section we will obtain the gravity dual of $\mathcal{N} = 1$ super Yang-Mills theory in four dimensions, arising in the low-energy dynamics of D6-branes wrapped on S^3 in T^*S^3, starting from eight-dimensional gauged supergravity. Let us start with an ansatz for the metric that describes such deformation of the world-volume of the D6-brane

$$ds^2 = e^{2f} dx_{1,3}^2 + e^{2h} d\Omega_3 + dr^2 \,, \tag{3.1}$$

where $d\Omega_3$ is the metric of the unit three-sphere. As explained in the introduction, wrapping the branes on a curved cycle implies that the theory has to be twisted on the curved part.

The fields on the D6-branes transform under $\mathrm{SO}(1,6) \times \mathrm{SO}(3)_R$ as $(\mathbf{8}, \mathbf{2})$ for the fermions and $(\mathbf{1}, \mathbf{3})$ for the scalars, while the gauge field is a singlet under R-symmetry. When we wrap the D6-branes on a three-cycle, the symmetry group splits as $\mathrm{SO}(1,3) \times \mathrm{SO}(3) \times \mathrm{SO}(3)_R$, and we shall construct a diagonal $\mathrm{SO}(3)_D$ from the SO(3) of the cycle and the one of the R-symmetry (in other words, we mix the spin conection with the gauge connection, as explained above). It can be easily seen that the effect of the twisting is to preserve the vector fields but transforms the scalars in one forms on the curved surface, so we are left with a theory with no scalars fields in the infrared; besides four supercharges are preserved.

We will describe the S^3 as a SU(2) group manifold by means of the left invariant forms w^i,

$$\begin{aligned} w^1 &= \cos\phi \; d\theta + \sin\theta \sin\phi \; d\psi \,, \\ w^2 &= \sin\phi \; d\theta - \sin\theta \cos\phi \; d\psi \,, \\ w^3 &= d\phi + \cos\theta \; d\psi \,, \end{aligned} \tag{3.2}$$

satisfying

$$dw^i = \frac{1}{2}\epsilon^{ijk} w^j \wedge w^k \,, \tag{3.3}$$

in terms of which the metric of the unit sphere simply reads

$$d\Omega_3 = \frac{1}{4}\sum_{i=1}^{3}(w^i)^2 \,. \tag{3.4}$$

The twisting is achieved by turning on a non-abelian SO(3) gauge field given by the left invariant form of the three sphere,

$$A^i = -(2g)^{-1} w^i \,, \tag{3.5}$$

whose field strength

$$F^i = -(8g)^{-1}\epsilon^{ijk} w^j \wedge w^k \,, \tag{3.6}$$

trivially obeys the corresponding equation of motion. This correponds to a complete identification of the spin connection with the R-symmetry. In such case it is possible to get rid of the scalars L^i_α,

$$L^i_\alpha = \delta^i_\alpha \Longrightarrow P_{ij} = 0 \,, \qquad Q_{ij} = -g\epsilon_{ijk} A^k \,. \tag{3.7}$$

The T-tensor is drastically simplified to $T_{ij} = \delta_{ij}$, $T = 3$, and $\Theta_{ij} = \frac{1}{4}\delta_{ij}$. Supersymmetric configurations require the following projections in the parameter ϵ:

$$\gamma_{\underline{r}}\ \epsilon = -i\gamma_9\ \epsilon \,, \qquad \gamma_{ab}\ \epsilon = -\sigma^{ab}\epsilon \,, \tag{3.8}$$

where $a, b = \underline{\theta}, \underline{\phi}, \underline{\psi} \equiv 1, 2, 3$ are the directions along the three-sphere. These projection leave unbroken 1/8 of the original supersymmetries, that is, four supercharges. The first order BPS equations are,

$$\begin{aligned} f' &= \frac{1}{3}\Phi' = -\frac{1}{2g}e^{\Phi-2h} + \frac{g}{8}e^{-\Phi} \,, \\ h' &= \frac{3}{2g}e^{\Phi-2h} + \frac{g}{8}e^{-\Phi} \,. \end{aligned} \tag{3.9}$$

By simple inspection, we can quickly find a solution

$$e^{2\Phi} = \frac{g^2}{16}r^2 \,, \qquad e^{2h} = \frac{3}{4}r^2 \,. \tag{3.10}$$

Notice that the relation $\Phi' = 3f'$ is forced from the Ricci flatness of the corresponding eleven-dimensional solution. When uplifted to 11d, we obtain a Ricci flat solution of the form $M_4 \times Y_7$ where Y_7 is a cone whose base X_6 is an Einstein manifold with the topology of $S^3 \times \tilde{S}^3$,

$$ds^2 = dx_{1,3}^2 + dr^2 + \frac{r^2}{9}\left[(w^a)^2 + (\tilde{w}^a)^2 - w^a\tilde{w}^a\right], \qquad (3.11)$$

$\tilde{w}^a$ being the left invariant one forms associated with $\tilde{S}^3$. This metric coincides with the asymptotic at large r of the M-theory solution on a G_2 holonomy manifold studied in ref. [12], and we note that the solution is singular in the infrared. It is natural to try to obtain a solution where the singularity is absent. In this system we do not have further degrees of freedom to turn on, that could occasionally solve the singularity; this means that there must be other solutions to the BPS equations (3.9), such that, when uplifted to eleven dimensions, do not give place to singularities in the infrared.

We can define indeed a pair of functions, $u \equiv h + \Phi$ and $v \equiv 3h - \Phi$, the system simplifies to $e^u du = \frac{g^2}{12}e^v dv$, whose immediate solution is

$$e^u = \frac{g^2}{12}\left(e^v - \frac{a^3}{3^{3/2}}\right), \qquad (3.12)$$

a being a constant. There is an amusing change of variable

$$r(\rho) = \frac{(2g)^{1/2}}{3^{3/2}}\left(\rho^{3/2}\, {}_2F_1\left[-\frac{1}{2};\frac{1}{4},\frac{1}{2};\frac{a^3}{\rho^3}\right] - a^{3/2}\frac{\sqrt{\pi}\,\Gamma(3/4)}{\Gamma(1/4)}\right), \qquad (3.13)$$

where ${}_2F_1[a,b,c,z]$ is the hypergeometric function[2]

$${}_2F_1[a,b,c,z] = \sum_{m=0}^{\infty}\frac{(a)_m(b)_m}{(c)_m}\,\frac{z^m}{m!}, \qquad (3.14)$$

with $(a)_n = \Gamma(a+n)/\Gamma(a)$ the Pochhammer symbol, that allows us to find a solution to the BPS equations of the form

$$e^{2\Phi} = \frac{g^3}{216}\rho^3\left(1 - \frac{a^3}{\rho^3}\right)^{3/2}, \qquad e^{2h} = \frac{g}{18}\rho^3\left(1 - \frac{a^3}{\rho^3}\right)^{1/2}. \qquad (3.15)$$

When uplifted to 11 dimensions, we obtain

$$ds^2 = dx_{1,3}^2 + \frac{1}{(1 - a^3/\rho^3)}d\rho^2 + \frac{\rho^2}{12}(\tilde{w}^a)^2 + \frac{\rho^2}{9}\left(1 - \frac{a^3}{\rho^3}\right)\left[w^a - \frac{1}{2}\tilde{w}^a\right]^2. \qquad (3.16)$$

This is the metric of a G_2 holonomy manifold which is topologically $\mathbb{R}^4 \times S^3$, originally constructed in [18, 19]. It turns out from our results that it can be obtained by means of an uplifting to 11d of a solution of eight-dimensional gauged supergravity. This fits

[2]Notice that in our case, for $\rho \geq a$, it has a real variable $z \leq 1$ such that the change of variables has not branch cut discontinuity. The substracted constant in (3.13) just amounts to $r(a) = 0$.

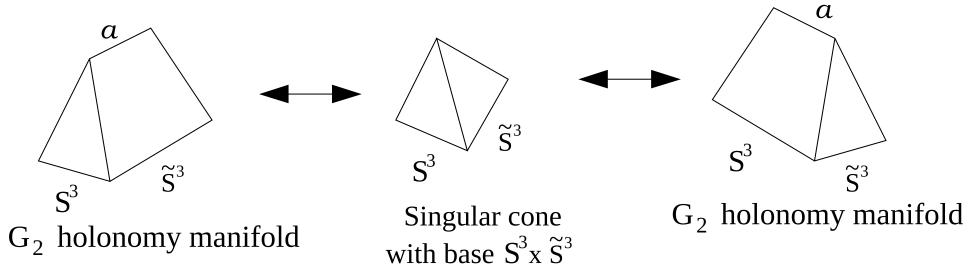

Figure 1: Flop transition in M-theory. The G_2 holonomy manifold on the left can be deformed to the one on the right. M-theory avoids the singular point due to the existence of C-field fluxes.

nicely with the discussions in [12], in the sense that one expects gauged supergravity to give a good description of the near horizon dynamics of D-branes, and this is what we are presently making manifest.

The solution obtained so far represents an M-theory background which is the direct product of four-dimensional Minkowski space and a G_2 holonomy manifold. In order to understand the physics behind this configuration we can go to type IIA. This issue was discussed in detail recently by Atiyah, Maldacena and Vafa [12], and we briefly remind here the main aspects of their discussion. The radial variable in (3.16) $\rho \geq a$ fills S^3 while the other sphere $\tilde{S}^3$ remains of finite volume a^3 when the former shrinks (see figure 1). The G_2 holonomy manifold has isometry group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_{\tilde{L}} \times \mathrm{SU}(2)_D$, the first two factors corresponding to the left action on S^3 and $\tilde{S}^3$, respectively, and the last one is the diagonal subgroup of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_{\tilde{R}}$. There is a flop transition in which the two spheres are exchanged. In this case, M-theory smooths out the singularity thanks to the existence of C-field fluxes through the three-sphere.

There are two very different quotients of this manifold: a singular one by modding out by $\mathbb{Z}_N \subset \mathrm{U}(1) \subset \mathrm{SU}(2)_L$, and a non-singular quotient if one instead chooses $\mathbb{Z}_N \subset \mathrm{U}(1) \subset \mathrm{SU}(2)_{\tilde{L}}$. This is due to the fact that S^3 shrinks to a point when $\rho \to a$ while $\tilde{S}^3$ has radius a. Modding out by $\mathbb{Z}_N \subset \mathrm{U}(1) \subset \mathrm{SU}(2)_L$ results in an A_{N-1} singularity fibered over $\tilde{S}^3$ so, after KK reduction along the circle corresponding to the U(1), one ends with N D6-branes wrapped on $\tilde{S}^3$ whose normal bundle is that corresponding to the three-sphere being a special lagrangian in the deformed conifold [12]. The holonomy gets correspondingly reduced from G_2 to SU(3) [15]. The second case, amounts to modding out by $\mathbb{Z}_N \subset \mathrm{U}(1) \subset \mathrm{SU}(2)_{\tilde{L}}$, which has no fixed points so the quotient is regular. After KK reduction one ends with a non-singular type-IIA configurations (without D-branes) on a space with the topology of $\mathcal{O}(-1) + \mathcal{O}(-1) \to \mathbb{P}^1$ [12], and with N units of RR flux through the finite radius S^2. This allowed the authors of ref. [12] to obtain the duality proposed by Vafa [10] from a geometrical flop in 11d.

The gauge theory defined in the planar directions of the D6-branes is, as we pointed out above, a four-dimensional $\mathcal{N}=1$ super Yang-Mills theory (possibly with extra light fields corresponding to KK modes). We will analize the gauge theory from a IIA perspective. The coupling constant of this gauge theory is given by

$$\frac{1}{g_4^2} = \frac{\mathrm{Vol}\,\tilde{S}^3}{g_7^2}\,. \tag{3.17}$$

Following [10], we can define in string theory a coupling superfield whose lower component is

$$Y = \frac{V}{g_s} + iC\,, \tag{3.18}$$

where V represents the volume of the three sphere in the string frame and C is the value of the three form potential of IIA that plays the role of the θ angle in the gauge theory. Besides, we introduce the usual gauge superfield $S = g_s\,\mathrm{Tr}\,W^2$, so that the action of the theory will be given by

$$\mathcal{S} = \frac{1}{g_s}\int d^2\theta\; Y S\,. \tag{3.19}$$

The instantons, that in our case will be euclidean D2-branes wrapped on the three-sphere, will correct this low energy action

$$\mathcal{S}_{\mathrm{inst}} \propto \int d^2\theta N^2 e^{Y/N}\,. \tag{3.20}$$

After extremizing with respect to the superfield Y and S, one obtains a superpotential where it can be seen that the breaking of U(1) to Z_2 has taken place, due to the appearence of N vacua and a gaugino condensate.

4. D6-branes on S^2 in T^*S^2

In the same vein of previous section, we now wrap N D6-branes on a holomorphic S^2 in T^*S^2. The corresponding split of the symmetry group is $\mathrm{SO}(1,4)\times\mathrm{SO}(2)\times\mathrm{SU}(2)_R$ such that, after twisting the SO(2) with a $\mathrm{U}(1)\subset\mathrm{SU}(2)_R$, the preserved fermions are those transforming in the $(4,\pm,\mp)$ representations, i.e. there are eight supercharges. Notice that out of three scalars, the one corresponding to the particular U(1) chosen in the R-symmetry group survives the twisting. This leaves us with the field content of $\mathcal{N}=2$ five-dimensional super Yang-Mills theory. The real scalar parametrizes the Coulomb branch of the theory.

From the gauged supergravity point of view, the twisting of this configuration allows us to get rid of all but one scalar field λ that enters L^i_α as

$$L^i_\alpha = \mathrm{diag}\left(e^\lambda, e^\lambda, e^{-2\lambda}\right), \tag{4.1}$$

such that the symmetric P_{ij} and antisymmetric Q_{ij} are

$$P_{ij} = \begin{pmatrix} \partial\lambda & 0 & -g\sinh 3\lambda\ A^2 \\ & \partial\lambda & g\sinh 3\lambda A^1 \\ & & -2\partial\lambda \end{pmatrix},$$

$$Q_{ij} = \begin{pmatrix} 0 & -gA^3 & g\cosh 3\lambda A^2 \\ & 0 & -g\cosh 3\lambda A^1 \\ & & 0 \end{pmatrix}. \tag{4.2}$$

It is enough to perform the twisting to turn on the gauge field A^3. The ansatz for the metric is

$$ds^2 = e^{2f} dx_{1,4}^2 + e^{2h} d\Omega_2 + dr^2 , \tag{4.3}$$

where $d\Omega_2 = d\theta^2 + \sin^2\theta d\phi^2$, and f and h are only functions of the radial coordinate. Imposition of the BPS conditions $\delta_\epsilon\psi_\mu = \delta_\epsilon\chi_i = 0$ translates into the following projections of the supersymmetric parameter

$$\gamma_{\underline{\theta\phi}}\ \epsilon = -\Gamma^{12}\ \epsilon , \qquad \gamma_{\underline{r}}\ \epsilon = -i\gamma_9\ \epsilon . \tag{4.4}$$

On the other hand, the BPS equations are:

$$f' = \frac{1}{3}\Phi' = -\frac{1}{6g} e^{\Phi-2\lambda-2h} + \frac{g}{24}\left(2e^{2\lambda} + e^{-4\lambda}\right) e^{-\Phi} , \tag{4.5}$$

$$h' = \frac{5}{6g} e^{\Phi-2\lambda-2h} + \frac{g}{24}\left(2e^{2\lambda} + e^{-4\lambda}\right) e^{-\Phi} , \tag{4.6}$$

$$\lambda' = \frac{1}{3g} e^{\Phi-2\lambda-2h} - \frac{g}{6}\left(e^{2\lambda} - e^{-4\lambda}\right) e^{-\Phi} . \tag{4.7}$$

Let us first present a solution in the simplest case of constant λ. It is clear from (4.7) that $\lambda > 0$ and $e^\Phi = \xi^{1/2} e^h$ with $\xi = \frac{g^2}{2}(e^{4\lambda} - e^{-2\lambda})$. Plugging this relation back into (4.5) and (4.6) it is immediate to obtain $\lambda = \frac{1}{6}\log\frac{3}{2}$ and $e^{2h} = \frac{3}{8}r^2$. Uplifting to 11d leads, after a suitable change of variables, to a metric of the form $M_5 \times Y_6$ where Y_6 is the singular conifold with base $T^{1,1}$,

$$ds^2 = dx_{1,4}^2 + dr^2 + r^2 d\Sigma_{1,1} ,$$

$$d\Sigma_{1,1} = \frac{1}{9}\left(d\psi + \sum_{a=1}^{2}\cos\theta_a d\phi_a\right)^2 + \frac{1}{6}\sum_{a=1}^{2}\left(d\theta_a^2 + \sin^2\theta_a d\phi_a^2\right) . \tag{4.8}$$

The fact that the solution in 11d involves a Calabi-Yau manifold is expected not only from the Ricci flatness condition but for the fact that, in 10d, the D6-branes are wrapping a holomorphic cycle of a K3 manifold whose holonomy is SU(2), and this uplifts to SU(3) holonomy manifolds [15]. It is natural to attempt a resolution of the singularity. The small resolution of the conifold has a metric given by [20, 21]

$$ds^2_{\text{res}} = \kappa(\rho)^{-1} d\rho^2 + \frac{1}{9}\kappa(\rho)\rho^2\left(d\psi + \sum_{a=1}^{2}\cos\theta_a d\phi_a\right)^2 + $$
$$+ \frac{1}{6}\rho^2\left(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2\right) + \frac{1}{6}\left(\rho^2 + 6a^2\right)\left(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2\right) , \tag{4.9}$$

where

$$\kappa(\rho) \equiv \frac{\rho^2 + 9a^2}{\rho^2 + 6a^2} \,. \tag{4.10}$$

When $a \to 0$ the metric reduces to that of the standard conifold (4.8). It is easy to see from (4.9) that near the former apex of the conifold the S^3 shrinks to zero while the S^2 remains of finite size a^2. To resolve the singularity, we should excite new degrees of freedom. It is natural, in this case, to look for excitations of the scalar field λ, that must be the source of the resolution factor κ. In the singular limit, as well as for large radius, $\kappa \to 1$, so it should enter as a logarithm, say,

$$\lambda(r) = \frac{1}{6}\left(\log\frac{3}{2} - \log\kappa(r)\right) . \tag{4.11}$$

We now insert this function back in (4.5), (4.6) and (4.7) and peform a not less amusing change of variables,

$$r(\rho) = \frac{2^{17/12}}{3^{17/12}}\rho^{3/2} F_1\left[\frac{3}{4}; -\frac{5}{12}, \frac{5}{12}; \frac{7}{4}; -\frac{\rho^2}{6a^2}, -\frac{\rho^2}{9a^2}\right] , \tag{4.12}$$

where $F_1[a; b_1, b_2; c; x, y]$ is the Appell hypergeometric function of two variables[3]

$$F_1[a; b_1, b_2; c; x, y] = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(b_1)_m(b_2)_n}{m!n!(c)_{m+n}}\, x^m y^n \,, \tag{4.13}$$

finding a solution to the BPS equations of the form

$$e^{2\Phi} = \frac{g^3}{144}\rho^3\kappa^{1/2}(\rho) \,, \qquad e^{2h} = \frac{3^{1/3}g}{2^{1/3}36}\rho(\rho^2 + 6a^2)\kappa^{1/6}(\rho) \,. \tag{4.14}$$

Uplifting of this solution to 11d gives precisely the metric (4.9) of the small resolution of the conifold $\mathcal{O}(-1) + \mathcal{O}(-1) \to \mathbb{P}^1$. It is again remarkable that this solution was extracted cleanly from eight-dimensional gauged supergravity. In this case, the radial coordinate $\rho \geq 0$ fills the three-sphere while the two-sphere remains of finite radius when $\rho \to 0$. The ALE A_{N-1} singularity fibered over S^2, that corresponds to N D6-branes (after KK reduction along the appropriate U(1)) in type-IIA wrapping an S^2 in T^*S^2, is obtained after modding out by $\mathbb{Z}_N \subset \mathcal{O}(-1) + \mathcal{O}(-1)$.

It is natural to ask at this point about the possibility of geometrical transitions of the $\mathcal{O}(-1) + \mathcal{O}(-1) \to \mathbb{P}^1$ manifold taking place. There are some key differences with respect to the configuration of the previous section. Indeed, the moduli space is

[3] Notice that the variables of the Appel function are real and negative so this change of variables is not singular neither it has branch cut discontinuities.

one (real) dimensional, being basically parametrized by the size of the sphere, that is, a. Thus, there is no possible conifold transition. Besides, in the present case, there is no way to avoid the singular point $a = 0$. This is related to the fact that these theories do not possess a θ term. This further prevents geometrical transitions of the sort discussed in [12, section 2] (in the context of type IIA). Moreover, when the system heads the singularity, the gauge theory is flowing towards the ultraviolet. There is no decoupling between the gauge theory and the string modes.

5. Conclusions

In the present paper we have obtained supergravity duals of D6-branes wrapping holomorphic two-cycles in local T^*S^2 and special lagrangian submanifolds in a deformed conifold. More concretely, we focused on the M-theory description of such configurations looking for geometric transitions that might be pointing in the direction of new superstring dualities that amount to large-N dualities of supersymmetric gauge theories.

It is interesting to remark that both metrics found in our paper, that is, the small resolution of the conifold and the G_2 holonomy manifold, are obtained from eight-dimensional supergravity, which is in some sense the *natural* theory for the near horizon dynamics of the D6-branes. Notice that our solutions correspond to the infinite coupling limit of type IIA, due to the fact that our 10d dilaton diverges for large values of the radial coordinates. Definitively, it would be of great interest to seek for solutions where the string coupling goes to a constant. Seemingly, our formalism is appropriate to address this issue. Besides, after this paper was completed, another interesting approach considering a similar problem in the case of Spin(7) holonomy manifolds was proposed [22].

Manifolds with G_2 holonomy give the appropriate M-theoretic background corresponding to $\mathcal{N} = 1$ four-dimensional theories. It is then of great interest to extend our results to include the other manifolds of this kind that were reported in the literature [18, 19]. From the string theory point of view, it has been recently argued that they correspond to the uplifting of space-time filling intersections of D6-branes [23].

The framework provided by this sort of dualities that emerge from geometrical transitions in M-theory seems very promising. In the last few months, interesting papers addressing issues such as confining string [24], domain walls [25], gluino condensation and remarkable dualities between different $\mathcal{N} = 1$ gauge theories [26] have appeared. The information provided by supergravity duals of these configurations is, in some sense, complementary to the above mentioned dualities: unlike the latters, the formers carry information of D terms.

We think that gauged supergravities provide a natural framework to pose some of these questions. Clearly, further research in the subject is deserved.

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